Nuclei

1. Using a nuclear counter the count rate of emitted particles from a radioactive source is measured. At $t = 0$ it was 1600 counts per second and $t = 8$ seconds it was 100 counts per second. The count rate observed, as counts per second, at $t = 6$ seconds is

2. The ratio of the mass densities of nuclei of ^{40}Ca and ^{16}O is

3. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is

4. A radioactive element decays by β emission. A detector records n beta particles in 2 second and in next 2 seconds it records 0.75 n beta particles. Find mean life (in second) corrected to nearest whole number. Given $\ell n2 = 0.6931$ and n 3 =1.0986.

5. The half life of radon is 3.8 days. After how many days will only one twentieth of radon sample be left over?

6. The count rate from a radioactive sample falls from 4.0×10^6 per second to 1×10^6 per second in 20 hour. What will be the count rate per second 100 hour after the beginning?

7. In an ore containing uranium, the ratio of U^{238} to Pb^{206} nuclei is 3. Calculate the age of the ore, (in year) assuming that all the lead present in the ore in the final stable product of U^{238}. Take the half life of U^{238} to be 4.5×10^9 year.

8. In a nuclear reactor U^{235} undergoes fission liberating 200 MeV of energy. The reactor has 10% efficiency and produces 1000 MW power. If the reactor is to function for 10 year, find the total mass (in kg) of the uranium required.

9. A radioactive source, in the form of a metallic sphere of radius 10^{-2} m emits β-particles at the rate of 5×10^{10} particles per second. The source is electrically insulated. How long (in second) will it take for its potential to be raised by 2 V, assuming that 40% of the emitted β-particles escapes the sources?

10. It is proposed to use the nuclear fusion reaction

$$_1H^2 + {_1H^2} \rightarrow 2He^4$$

in a nuclear reactor of 200 MW rating. If the energy from the above reaction is used with 25% efficiency in the reactor, how many gram of deuterium fuel will be needed per day? (The masses of $_1H^2$ and $_2He^4$ are 2.0141 amu and 4.0026 amu respectively).

11. A ^{7}Li target is bombarded with a proton beam current of 10^{-4} A for 1 hour to produce ^{7}Be of activity 1.8×10^8 disintegrations per second. Assuming that one ^{7}Be radioactive nucleus is produced by bombarding 1000 protons, determine its half life (in second)

12. The disintegration rate of a certain radioactive sample at any instant is 4750 disintegrations per minute. Five minutes later the rate becomes 2700 disintegrations per minute. Calculate half life of the sample (in minute)

13. If the radius of a nucleus ^{256}X is 8 fermi, then the radius (in fermi) of ^{4}He nucleus will be

14. The mass defect for the nucleus of helium is 0.0303 a.m.u. What is the binding energy per nucleon for helium in MeV ?

15. The radius of germanium (Ge) nuclide is measured to be twice the radius of $^9_4$Be. The number of nucleons in Ge are

SOLUTIONS

1. **(200)** According to question, at t = 0, $A_0 = \frac{dN}{dt} = 1600$ C/s

and at t = 8s, A = 100 C/s

$\therefore \frac{A}{A_0} = \left(\frac{1}{16}\right)$

Therefore half life period, $t_{1/2} = 2s$

$\therefore$ Activity at t = 6s = $1600\left(\frac{1}{2}\right)^3 = 200\,C/s$

2. **(1)** Nuclear density is independent of atomic number.

3. **(1)** We know that , $\left|\frac{dN}{dt}\right| = \lambda N = \frac{1}{T_{mean}} N$

$\therefore 10^{10} = \frac{1}{10^9} \times N$

$\therefore N = 10^{19}$

i.e. 10^{19} radioactive atoms are present in the freshly prepared sample.

The mass of the sample = $10^{19} \times 10^{-25}$ kg = 10^{-6} kg = 1 mg.

4. **(6.9)** We know that,

$N = N_0 e^{-\lambda t}$

After 2 second, $N_1 = N_0 e^{-\lambda \times 2}$

After (2 + 2) second, $N_2 = N_0 e^{-\lambda \times 4}$

According to given conditions,

$N_0 - N_1 = n$

or $N_0 - N_0 e^{-2\lambda} = n$ ... (i)

and $N_0 e^{-2\lambda} - N_0 e^{-4\lambda} = 0.75\,n$ ... (ii)

After solving above equations, we get

$\lambda = 0.145$ s

Mean life $T = 1/\lambda = 6.9$ s .

5. **(16.45)** Disintegration constant

$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{3.8} = 0.182$ per day

The number of particles left after time t

$N = N_0 e^{-\lambda t}$

or $\frac{N_0}{20} = N_0 e^{-\lambda t}$

or $e^{\lambda t} = 20$

or $t = \frac{\ln 20}{\lambda}$

$= \frac{\ln 20}{0.182} = 16.45$ days

6. **(3.91×10^3)** If A_0 is the initial activity of radioactive sample, then activity at any time

$A = A_0 e^{-\lambda t}$

or $1 \times 10^6 = 4 \times 10^6 \, e^{-\lambda \times 20}$

or $e^{-20\lambda} = \dfrac{1}{4}$

The count rate after 100 hour is given by

$A' = A_0 e^{-\lambda \times 100} = A_0 e^{-100\lambda}$

$= A_0 [e^{-20\lambda}]^5$

$= 4 \times 10^6 \left[\dfrac{1}{4}\right]^5$

$= 3.91 \times 10^3$ per second

7. **(1.868×10^9)** Suppose x is the number of Pb^{206} nulei. The number of U^{238} nuclei will be 3x, Thus

$3x + x = N_0$

We know that $N = N_0 e^{-\lambda t}$

or $3x = 4x e^{-\lambda t}$

$\therefore \; e^{\lambda t} = \dfrac{4}{3}$

or $t = \dfrac{\ln 4/3}{\lambda} = \dfrac{\ln 4/3}{(0.693/t_{1/2})}$

$= \dfrac{\ln 4/3}{(0.693/4.5 \times 10^9)}$

$= 1.868 \times 10^9$ year.

8. **(3.8×10^4)** If m kg is the required mass of the uranium, then number of nuclei

$= \dfrac{(m \times 1000) \times 6.02 \times 10^{23}}{235}$

Each U_{235} nucleus releases energy 200 MeV,

$\therefore$ total energy released in 10 years

$E_{in} = \dfrac{m \times 6.02 \times 10^{26}}{235} \times 200$

Energy required in 10 years, $E_{our} = Pt$

$= (1000 \times 10^6) \times (10 \times 365 \times 24 \times 3600)$

Efficiency $\eta = \dfrac{E_{out}}{E_{in}}$

Substituting the values, we get

$m = 3.8 \times 10^4$ kg.

9. **(6.94×10^{-4})** In time t, the total number of β-particles emitted $= 5 \times 10^{10}\, t$. As only 40% escape from the surface, so $N = 0.40 \times (5 \times 10^{10})t = 2 \times 10^{10}t$. The charge develops due to escape of each β- particle is 1.6×10^{-19} C, so total charge escapes in time t, $q = (2 \times 10^{10})t \times (1.6 \times 10^{-19})\, C$. Thus we can write,

$$V = \frac{1}{4\pi \epsilon_0} \frac{q}{R}$$

$$\text{or } 2 = \frac{1}{4\pi \epsilon_0} \frac{(2\times10^{10})t \times (1.6\times10^{-19})}{10^{-2}}$$

$\therefore t = 6.94 \times 10^{-4}$s

10. **(120)** Total energy used in nuclear reactor per day

$E = Pt$

$= (200 \times 10^6) \times (24 \times 3600)$

$= 1.72 \times 10^{12}$ J

Energy used in reactor per reaction

$$= \frac{25}{100}(2\times2.0141 - 4.0026)\times931\, MeV$$

$= 5.9584$ MeV

$= 9.5334 \times 10^{-13}$ J

The mass of deuterium used per reaction

$= 2 \times 2.0141 = 4.0282$ amu

$$= \frac{4.0282}{6.02\times10^{23}} g = 0.6691\times10^{-23} g$$

Thus mass of deuterium required to produce 1.72×10^{12} J of energy

$$= \frac{(0.6691\times10^{-23})\times(1.72\times10^{12})}{9.5334\times10^{-13}}$$

$= 120$ g.

11. **(8.63×10^6)** The total number of protons bombarded

$$= \frac{it}{\lambda} = \frac{10^{-4} \times 3600}{1.6 \times 10^{-19}} = 22.5 \times 10^{17}$$

Number of ^{7}Be produced

$$N = \frac{22.5 \times 10^{17}}{1000} = 22.5 \times 10^{14}$$

We know that activity

$A = \lambda N$

$$\text{or } A = \left(\frac{0.693}{t_{1/2}}\right) N$$

$$\therefore t_{1/2} = 0.693 \frac{N}{A}$$

$$= 0.693 \times \frac{22.5 \times 10^{14}}{1.8 \times 10^8}$$

$= 8.63 \times 10^6$ s

12. **(6.1)** We know that the rate of integration $\left|-\frac{dN}{dt}\right| = \text{A}$

$\therefore \text{A} = A_0 e^{-\lambda t}$

or $2700 = 4750 e^{-\lambda \times 5}$

or $\lambda = 0.1131$ per minute

$$\text{Half life } t_{1/2} = \frac{0.693}{\lambda}$$

$$= \frac{0.693}{0.1131} = 6.1 \text{ minute}$$

13. **(2)** $\text{R} = \text{R}_0(\text{A})^{1/3}$

$$\therefore \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{256}{4}\right)^{1/3} = 4$$

$$R_2 = \frac{R_1}{4} = 2 \text{ fermi}$$

14. **(7)** $\frac{\text{Binding energy}}{\text{Nucleon}} = \frac{0.0303 \times 931}{4} \approx 7$

15. **(72)** We use the formula,

$R = R_0 A^{1/3}$

This represents relation between atomic mass and radius of the nucleus.

For berillium, $R_1 = R_0 (9)^{1/3}$

For germanium, $R_2 = R_0 A^{1/3}$

$$\frac{R_1}{R_2} = \frac{(9)^{1/3}}{(A)^{1/3}} \Rightarrow \frac{1}{2} = \frac{(9)^{1/3}}{(A)^{1/3}}$$

$$\Rightarrow \frac{1}{8} = \frac{9}{A} \Rightarrow A = 8 \times 9 = 72.$$